الدوال العكسية

- **properties of functions:**

① $\begin{cases} y = 2x + 5 \longrightarrow \text{Relation} \\ x^2 + y^2 = 4 \longrightarrow \text{Function} \end{cases}$

(each vertical line intersect with function in one point)
(each value of x have one value of y).

→ Is every function have inverse?

x is function of y (when each value of x has one and it is converse value of y).

② or $\boxed{\begin{array}{l} y = x^2 \ (\text{not inv.}) \\ y = 2x+5 \ (\text{inv.}) \end{array}}$ (each horizontal line intersect curve in one point).

→ one to one function. (y is function of x)
(the only function that (x is function of y). } The condition
has inverse). } one to one

if $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$
$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ } definition.

⟹ From (1) $2x_1 + 5 = 2x_2 + 5$ ∴ $x_1 = x_2$ (one to one func.)
⟹ From (2) $x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$ ∴ $x_1 \neq x_2$ (not one to one

$y = x^2$ $(x \geq 0) \Rightarrow$ is one to one when there is a condition in specific interval.

→ The relation between function and inverse?
$y = f(x) \longrightarrow$ function.
$x = f^{-1}(y) \longrightarrow$ The Form of inverse function.
from (1) $f(x) = 2x + 5$ $x = f^{-1}(y) = \dfrac{y-5}{2}$
$f^{-1}(x) = \dfrac{x-5}{2}$

→ The Relation between function and inverse? (f and f^{-1})
- Domain of f is Range of f^{-1}
- Range of f is domain of f^{-1}.
- graph is symmetric about ($y=x$)

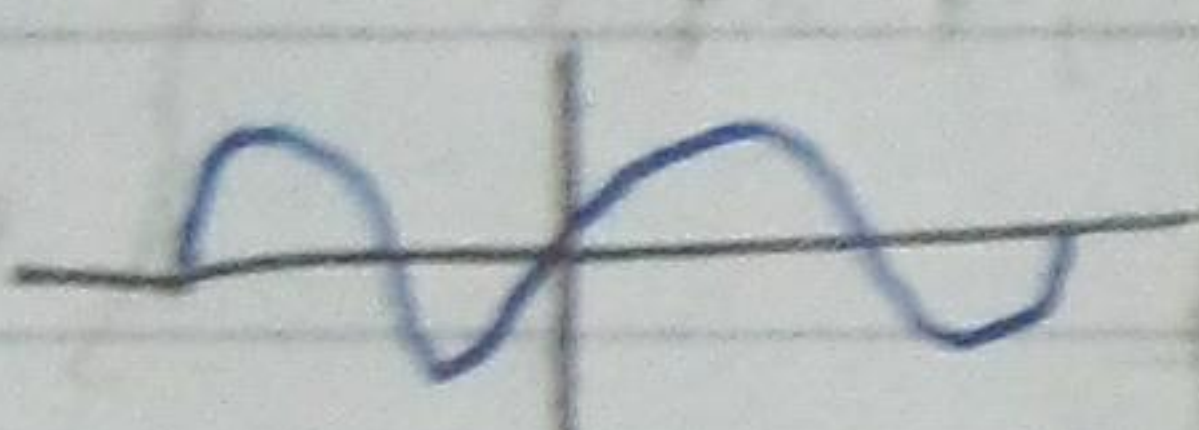$$f^{-1}(f(x)) = x \quad , \quad f(f^{-1}(x)) = x$$

- inverse function: is a function of (one to one function)
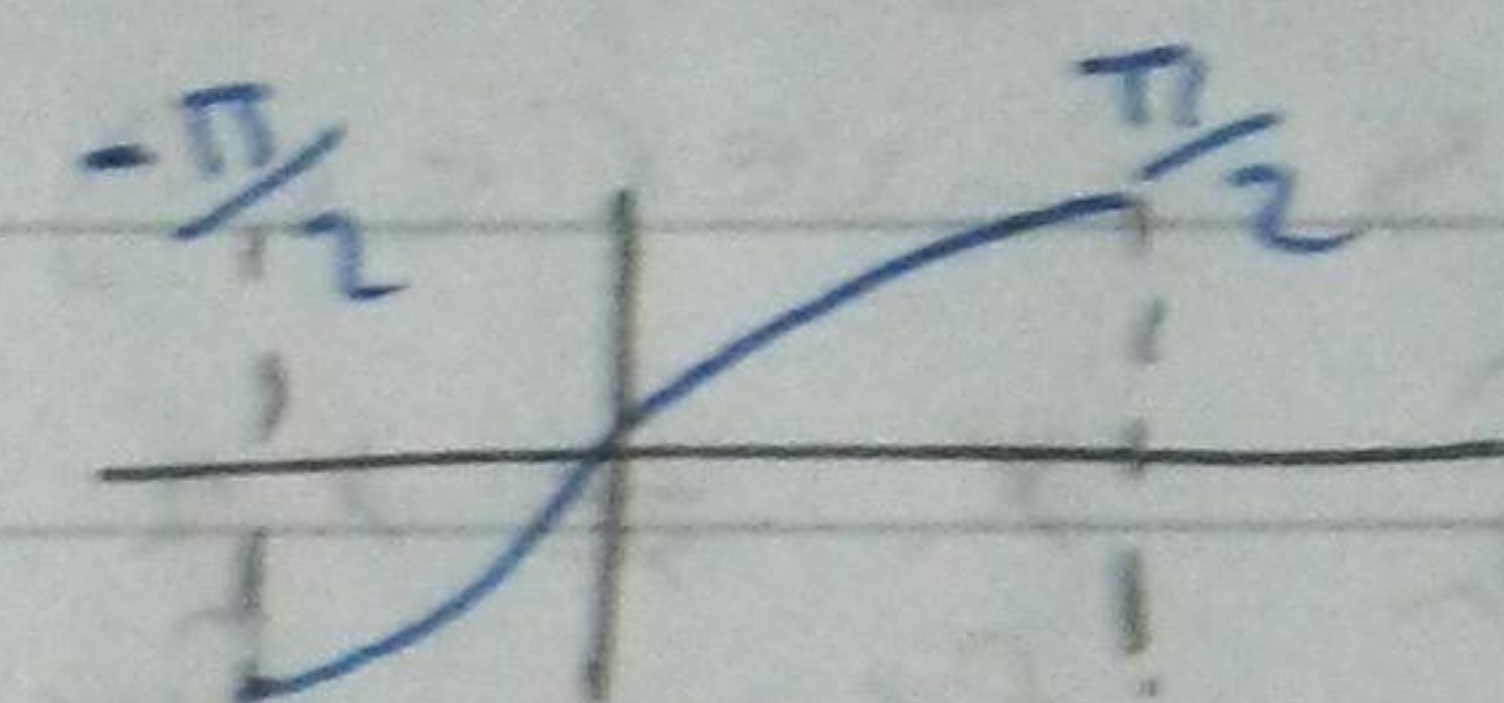→ if a function isn't one to one (There is an inverse function of specific interval).

- Inversed Triagometric functions:
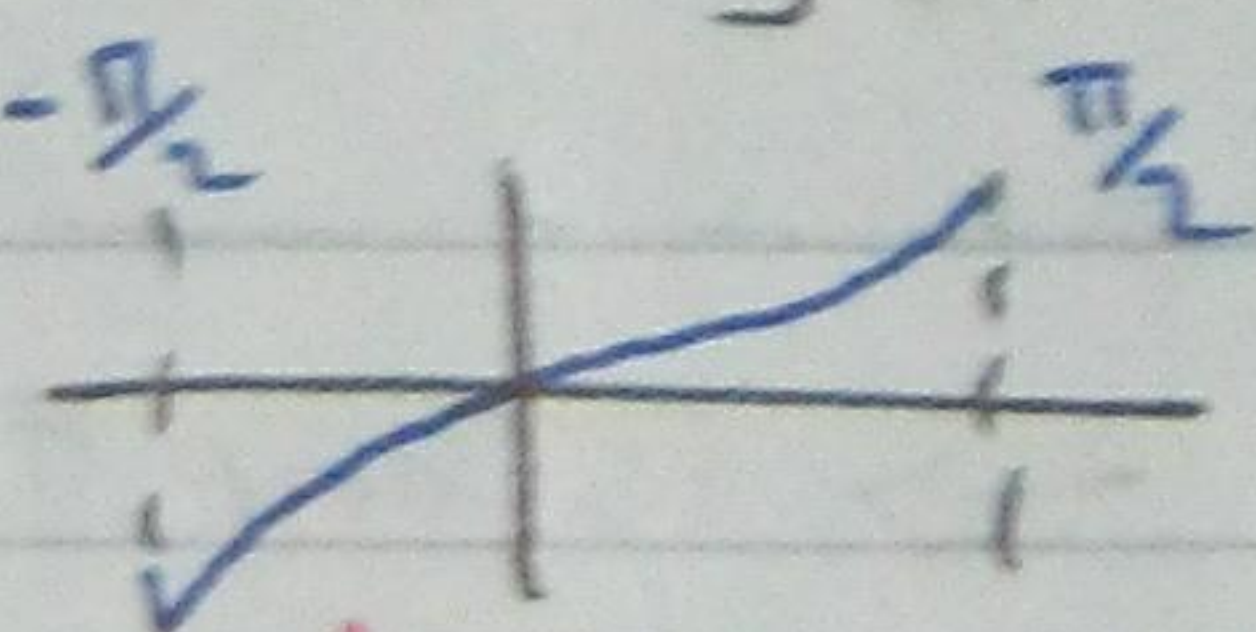$$y = \sin x \Rightarrow \text{inverse}: y = \sin^{-1} x \neq \frac{1}{\sin x} = (\sin x)^{-1}$$
Domain : R $\Rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$ to be one to one function.
Range : $[-1, 1]$

Graph:

Graph:
(not one to one)

③ $y = \sin^{-1} x \Rightarrow D: [-1, 1]$, Range $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
→ Graph:

→ Drevetive : $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

⇒ from ③
$$\sin y = \sin(\sin^{-1} x) = x$$
$$x = \sin y$$
$$\frac{dx}{dy} = \cos y = \sqrt{1 - \sin^2 y}$$
$$= \sqrt{1 - x^2}$$
$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - x^2}}$$

Drevetive : $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

Note
$$\frac{dy}{dx} = \left(\frac{1}{dx/dy}\right)$$

$$y = 2x + 1$$
$$\frac{dy}{dx} = 2$$
$$x = \frac{y-1}{2}$$
$$\frac{dx}{dy} = \frac{1}{2}$$
(complete drevetive)

$$\frac{d}{dx}(\sin^{-1}x) = \frac{1}{\sqrt{1-x^2}} \quad , \quad \frac{d}{dx}(\sin^{-1}u(x)) = \frac{u'(x)}{\sqrt{1-u^2(x)}}$$

if الـ

$\longrightarrow u'(x) \longrightarrow$ Drevetive of function but (u not u^2).

$$\sin x = \frac{1}{2} \qquad \qquad \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

as Range of $\sin^{-1}x$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$

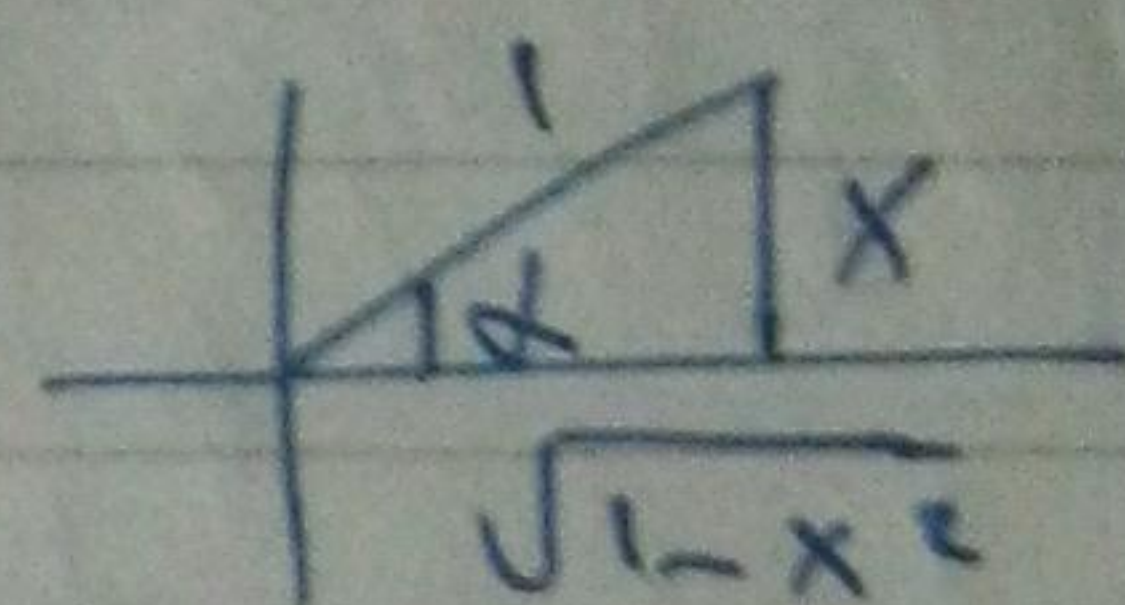$$\sin^{-1}\left(\frac{-1}{2}\right) = -\frac{\pi}{6}$$

Note : each angle in the
fourth quarter is (-ve).
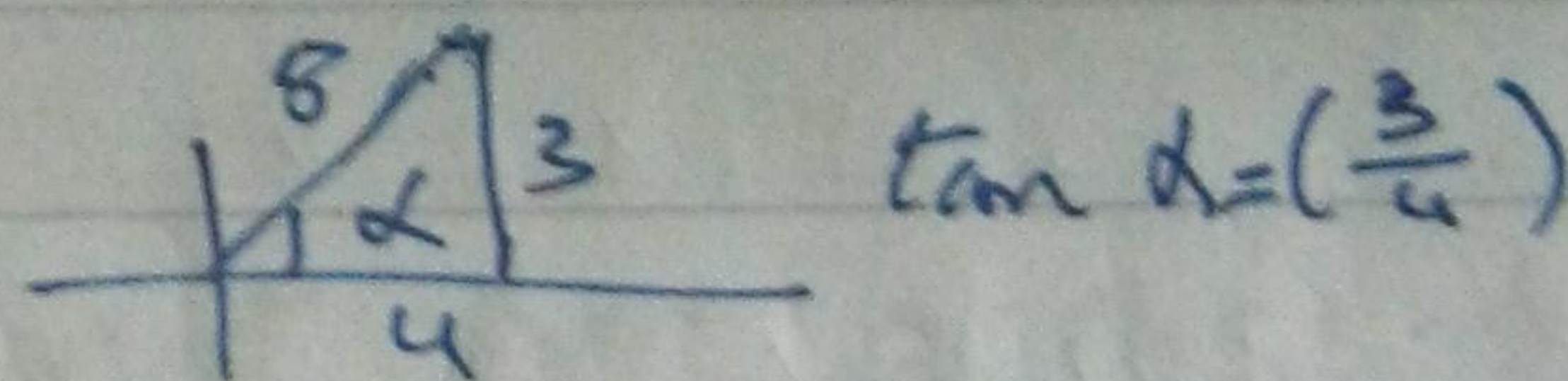as Domain Range $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. not $[0, 2\pi]$.
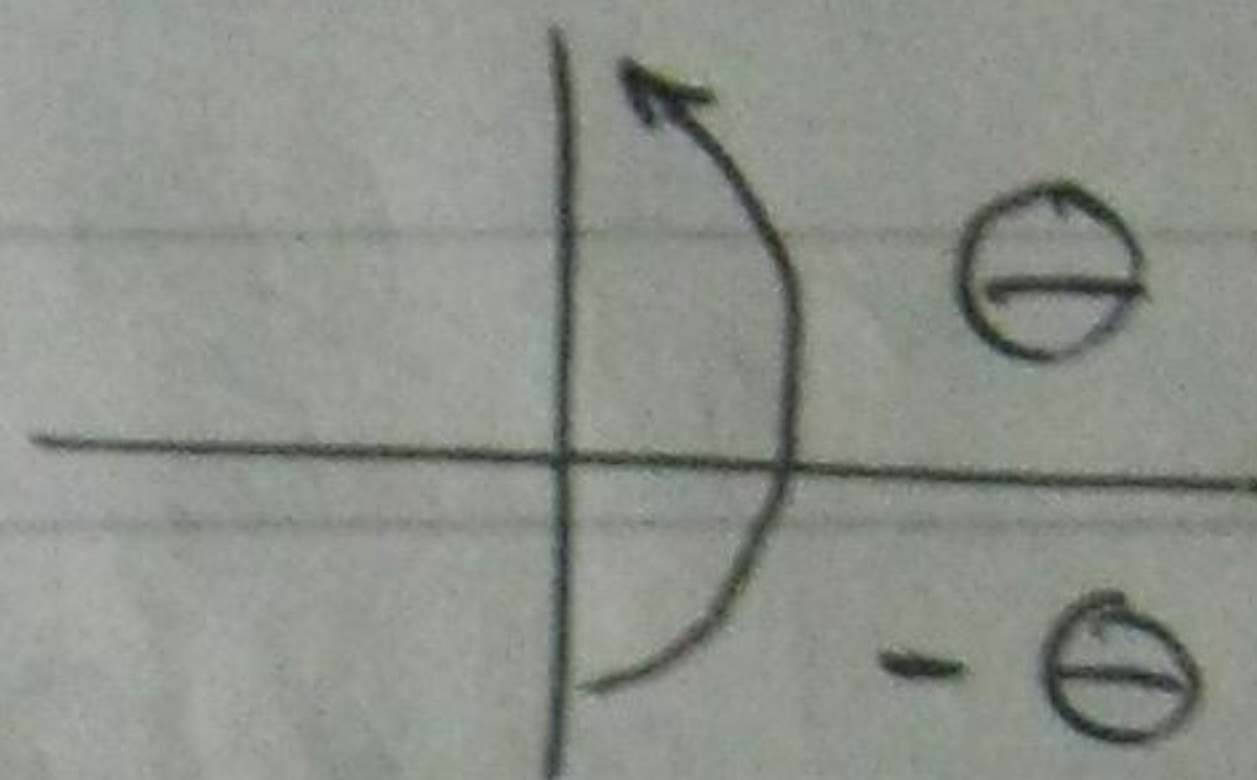
$$\cos(\sin^{-1}x) \Rightarrow \sin^{-1}x = \alpha$$
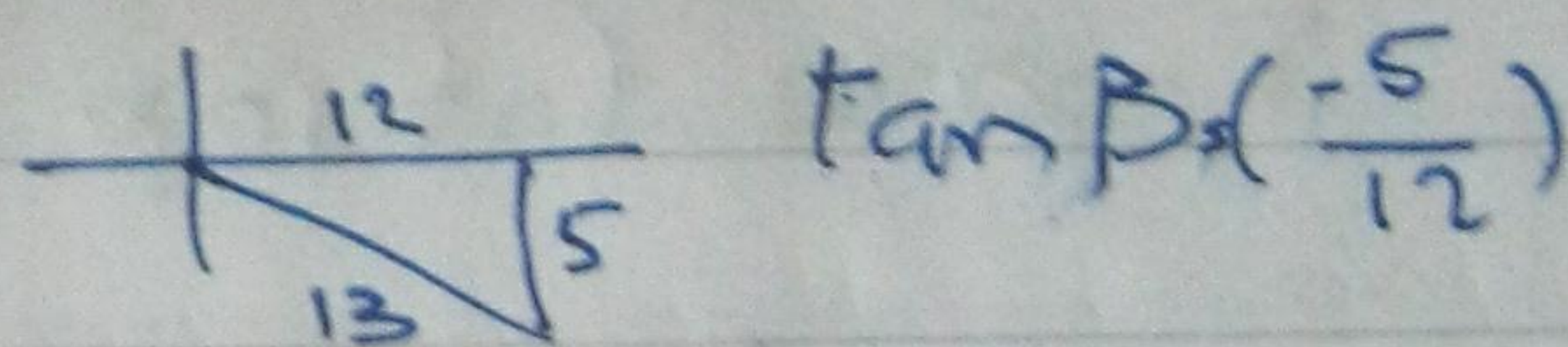$$x = \sin \alpha.$$

ex
$$\Rightarrow \tan\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\left(-\frac{5}{13}\right)\right)$$

$\alpha \qquad \qquad \beta$

1) $\sin^{-1}\frac{3}{5} = \alpha \qquad \sin \alpha = \frac{3}{5} \qquad \qquad \tan \alpha = \left(\frac{3}{4}\right)$

2) $\sin^{-1}\left(-\frac{5}{13}\right) = \beta \quad \sin \beta = \frac{-5}{13} \qquad \qquad \tan \beta = \left(\frac{-5}{12}\right)$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{\left(\frac{3}{4}\right) + \left(\frac{-5}{12}\right)}{1 - \left(\frac{3}{4}\right)\left(\frac{-5}{12}\right)}$$

ex
$$\Rightarrow y = \sin^{-1}(x^2)$$

$(x^2)^2 \Rightarrow$ The function squared

$$y' = \frac{2x}{\sqrt{1 - (x^2)^2}}$$

$(2x) \Rightarrow$ derevitive of function.
not function's squared

ex
$$y = \sin^{-1}(e^{2x}).$$
$$y' = \frac{2e^{2x}}{\sqrt{1 - (e^{2x})^2}}$$

ex
$$y = \sin^{-1}(\sqrt{x^2+1})$$
$$y' = \frac{\frac{2x}{2\sqrt{x^2+1}}}{\sqrt{1 - (x^2+1)}}$$

ex $\Rightarrow y = \sin^{-1}(\cos x)$.

$$y' = \frac{-\sin x}{\sqrt{1 - \cos^2 x}} =$$

$$= \frac{-\sin x}{\sin x} = \boxed{|-1|}$$